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Abstract

This paper studies the costs and benefits of delegating decisions to superiorly informed agents relative to the use of rigid, non discretionary contracts. The main focus of the paper lies in the analysis of the costs of delegation, primarily agency costs, versus their benefits, primarily the flexibility of the action choice.

We first determine and characterize the properties of the optimal flexible contract. We then show that the higher the agent's degree of risk aversion, the higher is the agency costs of delegation and the less profitable a flexible contract relative to a rigid one. When the parties to not have sharp probability beliefs, the agent's degree of imprecision aversion introduces another agency cost, which again reduces the relative profitability of flexible contracts.

JEL Classification: D86, D82, D81.
Keywords: Delegation, Flexibility, Agency Costs, Multiple Priors, Imprecision Aversion.

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1 Introduction

Motivation. A central problem in organizations is the fact that agents assigned a given task may end up having, at the time they have to act, some superior information on the suitability of the various actions which can be taken to perform the assigned task. As a consequence, it may be desirable, in order to enhance the performance of the organization, to grant agents some degree of discretion in their choice of which action to undertake, or to ask them to report their information before specifying which action should be carried out. The obvious difficulty in doing this is that the interests of such agents may not be aligned with those of the organization. This difficulty can be mitigated and possibly eliminated with the use of appropriate monetary transfers to the agents, that is of appropriate compensation contracts. For such contracts to work, some risk must be typically shifted to the agents. If agents are risk averse, doing this is costly. Moreover, if the nature of the possible realizations of the uncertainty, that is of the possible circumstances in which the actions might have to be taken and of their consequences, is not clearly understood a priori, either because some unforeseen contingencies may arise or because the probabilities of the possible events may be 'ambiguous', some further difficulties and costs arise.

The presence of these costs implies that, in the decision of whether or not and to which extent to delegate to an agent the choice of which action to undertake, a trade-off is faced. On the one hand, the wider the uncertainty concerning the environment in which the agent will have to take his action and the more important is for the organization the fact that the 'right' action is taken in each possible circumstance, the higher are the benefits of delegating the choice to the agent, that is of offering him a contract granting some flexibility in his choice. On the other hand, the extent and nature of this uncertainty also affect the costs of delegation, in a way which depends on the risk aversion of the agent, as well as on the degree of 'ambiguity' of such uncertainty and the attitude towards it exhibited by the agent. When this cost is sufficiently high it might be preferable to opt for a different type of contract, which does not delegate the action choice to the agent.

The issue is important as this trade-off naturally arises when the architecture of organizations is evaluated. The main focus of this paper is on the analysis of this trade-off, and in particular of how the cost of delegating decisions to superiorly informed agents varies with the structure of the uncertainty and the agents' attitude towards risk and uncertainty.

Model and results. To this end, we will consider a simple contracting situation between a principal and an agent. The agent must take a costly action which generates some revenue for the principal. Before taking his action, but after signing the contract, the agent receives a private signal over the productivity of the various actions. More precisely, we assume the agent privately learns the realization of a variable which, together with the action chosen by the agent, affects the probability of the different realizations of the principal's revenue. The action chosen by the agent is not observable by the principal but we suppose that, at the time of contracting, the principal has the ability to predefine the set of actions, or possible tasks, available to the agent. Thus the principal could specify a determinate action that the agent must undertake in all the possible circumstances he may have to act - what we will call a *rigid*, or non discretionary, contract. Alternatively, the principal could leave the agent some discretion in his behavior, so that the action the agent undertakes may vary with the information received - a *flexible* contract.

Also, the cost for the agent of undertaking the various actions is deterministic. Hence in the absence of monetary transfers contingent on the realization of the principal's revenue the interests of the principal and the agent are not aligned as the latter would always choose the least costly action among the ones available to him. A flexible contract must then include a suitably designed compensation scheme, which might also vary with the agent's report over the signal received, so as to induce him to take the revenue maximizing action for each realization of the signal. But such variability in the compensation generates possible agency costs. In contrast, a rigid contract is simpler, does not need to rely on high-powered incentives and never incurs any agency cost.

Consider first the case where principal and agent have common and sharp probabilistic beliefs over the possible events in which the agent will have to act. In this environment, if the agent is risk neutral[1], agency costs are zero and the optimal flexible contract always dominates, at least weakly, the rigid contract. This is no longer true if the agent is risk averse, as agency costs are positive in that case. We characterize the optimal flexible contract when the agent has CARA preferences so as to be able to isolate the effects of changes in the agent's risk aversion. We find that at the optimal flexible contract the agent's compensation also depends on the agent's report over the signal received and that the agent's utility is not equalized across different realizations of the signal.

Also, an increase in the agent's degree of (absolute) risk aversion implies a larger agency cost, and hence a lower profitability for the principal of the optimal flexible contract relative to the rigid contracts. Thus, there is a threshold level for the agent's degree of risk aversion, above which a rigid contract always dominates the flexible one and below which the reverse is true. On the other hand, the effects of increasing risk aversion on the form of the incentive contract, for instance on the variability in the compensation paid to the agent across different realizations of the output, prove more sensitive to changes

[1] We assume the principal is always risk neutral.

in the parameters of the environment. The benefits of the flexible contract are then larger the greater is the variance of the productivity of the various actions the agent may undertake across the different realizations of the signal, that is the greater the relevance of the information received by the agent.

We turn then our attention to situations where the information available to the parties concerning the possible events in which the agent will have to act is not precise enough to pin down a single probability distribution. This might be for instance because the circumstances under which the agent finds himself to operate are totally new, with almost no information available. Or it might capture the fact that these events are hard to describe precisely in full details. We model this fact by assuming that principal and agent have a common *set* of probabilistic beliefs over the likelihood of these events and allowing them to have possibly different degrees of imprecision aversion.[2] We actually assume the Principal is imprecision neutral while the Agent is imprecision averse. To contrast this with the situation under risk aversion, we assume both parties are risk neutral and show that imprecision aversion by itself creates an agency cost. We provide then a partial characterization of the optimal flexible contract under imprecision aversion, showing the properties of the optimal flexible contract in this case are different from those obtained under risk aversion. We also show that increasing the agent's imprecision aversion reduces the profits at the optimal flexible contract, making so the rigid contract more attractive.

Even though, with multiple priors the compensation contract may be designed in such a way that principal and agent end up "using different beliefs", and hence possibly engage in mutually beneficial speculative trade, we show this is never optimal. This stands in contrast with the case in which both principal and agent have sharp, but different prior beliefs, where the surplus generated by the contractual relationship is actually enhanced by the possibility of exploiting the benefits of speculative trade (as in Eliaz and Spiegler (2007)).

Literature.

The choice in organizations between flexible and rigid contracts has been examined in various other papers. Most of them however focused on the case where, in contrast to the setup considered here, monetary transfers are not allowed and the objectives of principal and agent are at least partly aligned. In such environments the agent may be willing to freely transmit some of his private information to the principal. Dessein (2002) investigates the trade-off between contracts where the choice of the action is delegated to the agent and contracts where the principal retains the control over such choice, but uses

[2]We follow here the model of Gajdos, Hayashi, Tallon, and Vergnaud (2008).

the information that is reported to him by the agent. He examines in particular how such trade-off varies with the degree of congruence between the objectives of the principal and the agent. Both Aghion and Tirole (1997) and Szalay (2005) study the consequences of delegating to the agent - possibly only in part - control over the action choice on the same agent's incentives to invest in acquiring information.

Probably the closest paper to ours in this literature is Prendergast (2002). He considers an environment where, like in ours, monetary transfers are allowed, the structure of information is given and the agent has superior information. He also examines how the relative benefits of flexible and rigid contracts vary, but with respect to the magnitude of the uncertainty facing the agent, that is the variability in the possible situations in which he may find himself to act. Prendergast considers the case where the agent is risk neutral and agency costs are exogenously given (as fixed 'monitoring costs'). On the other hand our main focus here, as argued above, is on the endogenous determination of such costs and the analysis of how they vary with the agent's attitude to uncertainty and the precision of the information of principal and agent concerning the uncertainty they face in the contractual design.

A rather different characterization of the trade-off between rigidity and flexibility is provided by Hart and Moore (2008), where the main cost of delegation lies in the variability of the outcome prescribed by the contract and the deadweight losses this generates.

The effects of ambiguity or imprecision in the probabilistic beliefs concerning the possible realizations of the environment faced by parties in contractual situations have been first examined by Mukerji (1998) and Ghirardato (1994). Mukerji (1998) studies a vertical relationship problem, using the Choquet expected utility model of Schmeidler (1989). He shows that, as a result of ambiguity aversion, the optimal contract might be incomplete and, differently from our setup, exhibit low powered incentives. Ghirardato (1994) looks at a standard moral hazard problem but where parties' "beliefs" are non-additive, reflecting uncertainty aversion: each action taken by the agent induces a non-additive distribution on outcomes. His results are not directly comparable with ours, in particular because of the use of different underlying decision models. He can show in some very particular case that a decrease in the degree of non additivity (i.e., of imprecision in our setup) will not decrease the principal's profits.

The paper is organized as follows. The next section describes the environment while Section 3 presents the contracting problem, studies its solution and characterizes it. Section 4 then studies the trade-off between flexible and rigid contracts, how the choice of delegation varies with different features of the environment, in particular the agent's attitude towards risk. In the final section we consider the situation where the parties do not have sharp probability beliefs and investigate how the agent's degree of ambiguity aversion affects the choice between flexible and rigid contracts.

2 The set-up

We consider a contractual relationship between a principal, say a firm, and an agent, say a worker. The worker has two possible actions, x and y. The output generated by each action is uncertain: it can be either high ($\bar{R}$) or low ($\underline{R}$). The probability of the different output realizations when action x (resp. y) is undertaken is also uncertain and depends on some event $\theta \in \{\theta_1, \theta_2\}$: it is $\pi(x, \theta)$ (resp. $\pi(y, \theta)$) for $R = \bar{R}$.

The realization of the output is publicly observable while the action chosen by the agent is only privately known by him. Furthermore the realization of θ, describing possible events affecting the execution/profitability of the different possible actions, is privately observed by the agent before his action is chosen, not by the principal (nor by any third party). To begin with, we examine the case where both principal and agent have sufficient information over the generating process of this uncertainty to come up with a sharp probabilistic belief over it: let p denote their common belief concerning the occurrence of θ_1.

The contract is written before the realization of any source of uncertainty (i.e., before the output and θ are realized). Although the action undertaken by the agent is not observable, we assume that, at the time the contract is signed, the principal can impose some restrictions over the set of actions available to the agent.[3] To understand the nature of these restrictions we can think, for instance, of a situation where the principal can decide to install either only one software on the agent's computer (in which case only one action is available to the agent) or different types of software. In the latter case the agent is free to choose which software to use (x or y) to perform the task and his actual choice is not observable. Also, the fact that such restrictions can only be imposed ex ante can be justified if we think of situations where the timing of the resolution of the uncertainty over θ, and hence of the action choice, is also uncertain and privately observed.

In this framework, therefore a compensation contract is a specification of a set of admissible actions $A \subseteq \{x, y\}$ together with a wage payment w from the principal to the agent, where w can depend on the realized level of the output and the agent's announce-

[3]The possibility of imposing such restrictions was earlier considered in various papers starting with Holmstrom (1984) (see Alonso and Matouschek (2008), Armstrong and Vickers (2009) for some recent contributions). This was however typically in the absence of monetary transfers. In addition, in this literature the action is typically observable.

ment about the realization of the event θ. Let $\bar{w}_i$ (resp. $\underline{w}_i$) denote the compensation paid to the agent when the output is $\bar{R}$ (resp. $\underline{R}$) and the (declared) state is $\theta_i, i = 1, 2$.

In particular, we would like to distinguish the case where the full menu of possible actions is available to the agent, $A = \{x, y\}$, from the cases where only action x - or only action y - is available to the agent. We refer to the contract in the first case as a *flexible contract*, since the agent has the flexibility and the discretion to choose the action he thinks is more appropriate for him (and suitable incentives should be specified in the contract to induce the agent to make a choice also in the principal's interest). In the second case we say on the other hand the contract is *rigid*, as it prescribes the agent to always undertake a given action. The contract can then be of type x or of type y according to which action is specified.

The time-line is then as follows:

$t = 0$ The contract is signed, specifying the payments due to the agent for each possible realization of the output and each announcement of the agent regarding θ. In addition, the contract specifies the set $A \subseteq \{x, y\}$ of possible actions available to the agent.

$t = 1$ θ is observed by the agent who announces then its value to the principal.

$t = 2$ The agent undertakes an action $z \in A$, not observable by the principal.

$t = 3$ Output is revealed (i.e., uncertainty about output is resolved and output is observed)

$t = 4$ Compensation is paid to the agent, according to the realized output level and the agent's announcement.

Observe that at the time in which the contract is signed there is symmetric information among the parties, the agent does not know the realization of the uncertainty. Asymmetric information will arise at a later stage, when the agent learns some information about the profitability of the different actions, and chooses then which action to take.

Remark 1 *We ignore here the possibility of renegotiation, in particular at the time in which the realization of θ is learnt by the agent* $(t = 1)$.

The principal is the residual claimant of the output and is risk neutral. His payoff, when action z_i, $i = 1, 2$, is implemented in state θ_i, is then given by the expected profit:

$$p[\pi(z_1, \theta_1)(\bar{R} - \bar{w}_1) + (1 - \pi(z_1, \theta_1))(\underline{R} - \underline{w}_1)] \\ +(1 - p)[\pi(z_2, \theta_2)(\bar{R} - \bar{w}_2) + (1 - \pi(z_2, \theta_2))(\underline{R} - \underline{w}_2)]$$

The agent has a non separable[4] utility function over the compensation received and the cost c_z of undertaking the action $z \in \{x, y\}$ that is chosen. In particular, in most of the paper we will assume the agent is risk averse and exhibits the following preferences:

Assumption 1 *The agent has a CARA utility function:* $u(w, z) = -\frac{e^{-a(w-c_z)}}{a}$, *with* $a > 0$.

The agent's risk attitude is so described by the single parameter a. It is then convenient to renormalize the agent's reservation utility as $-\frac{e^{-a\bar{u}}}{a}$.

Our main goal is to investigate in this set-up the relative profitability of flexible and rigid contracts. While the flexible contract offers the agent the opportunity to choose the best action in each possible contingency, delegating the choice to the agent creates an agency problem, since the action is not observable. Hence the wage schedule has to satisfy a set of appropriate incentive compatibility constraints. On the other hand, in a rigid contract no agency problem arises, since the agent has no discretion, but the action implemented cannot be adjusted to the different contingencies.

We will also assume:

Assumption 2

i) $c_x > c_y$, *i.e.,* $\Delta c \equiv c_x - c_y > 0$,

ii) $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_2) > \pi(y, \theta_1)$,

iii) $(\pi(x, \theta_1) - \pi(y, \theta_1))(\bar{R} - \underline{R}) > \Delta c > (\pi(x, \theta_2) - \pi(y, \theta_2))(\bar{R} - \underline{R})$,

iv) $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$.

Conditions i) and ii) say that action x is both more costly and more productive than action y. At the same time, the additional productivity of action x, relative to action y, is uncertain: it is larger in state θ_1 than in state θ_2.[5] Condition iii) then says that this variability in the productivity differential is sufficiently significant that in state θ_1 the expected revenue net of the cost is higher for action x and in state θ_2 it is higher for action y. Hence conditions i-iii) ensure that, if there were no agency problems (that is, if

[4] A utility function that is non separable in the wage received and the cost incurred allows us to study the comparative statics properties of the optimal contract with respect to the agent's level of risk aversion - one of our objectives. With such a specification in fact the rate of substitution between actions and wage payments is constant and changes in the curvature of the agent's utility function only capture changes in the agent's attitude towards risk in his compensation.

[5] Condition ii) specifies one such configuration. An alternative specification which may also exhibit this property is $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_1) > \pi(y, \theta_2)$; its effects for the form of the optimal contracts are discussed below in footnote 6.

both θ and the agent's action were publicly observed), the optimal contract would be a flexible one, implementing action x in θ_1 and y in θ_2.

Finally, condition iv) says that in state θ_1 the productivity differential of action x relative to y is sufficiently large, relative to the utility cost of effort. It ensures, as we will see, that the agency costs are not too high and hence that the profile of actions x in θ_1 and y in θ_2 is implementable even when the state θ and the agent's actions are only privately observed.

3 Contracts

3.1 Optimal flexible contract

The advantage of a flexible contract over a rigid one is that it allows to implement the action profile maximizing net revenue that, under Assumption 2, is given by action x in θ_1 and y in θ_2. The cost is that, to implement such action profile, appropriate incentive constraints need to be imposed, ensuring that no possible deviation, in the action choice and/or the reporting over the state, is profitable. The optimal contract implementing this action profile subject to the incentive constraints (to which we will refer, with a slight abuse of terminology, as the *optimal flexible contract*) is obtained as solution of the following programme:

$$
\begin{aligned}
\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\
& +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\
\text{s.t.} \quad & \\
& \left\{
\begin{array}{ll}
(IC1) & -\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}-(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \geq -\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)}-(1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \\
(IC2) & -\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}-(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \geq -\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}-(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \\
(IC3) & -\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}-(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \geq -\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}-(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\
(IC4) & -\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}-(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \geq -\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)}-(1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\
(IC5) & -\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}-(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \geq -\pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}-(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)} \\
(IC6) & -\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}-(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \geq -\pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)}-(1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)} \\
& \\
(PC) & -p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]- \\
& \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \geq -e^{-a\bar{u}}
\end{array}
\right.
\end{aligned}
\qquad (P^{flex})
$$

where incentive constraints (IC1),(IC2) and (IC3) ensure that, in state θ_1, the agent does not want to deviate by, respectively, misreporting the state, changing the action, or doing both. Incentive constraints (IC4), (IC5) and (IC6) ensure the same properties hold in state θ_2. (PC) is then the participation constraint.

We show in the next proposition that, at a solution of the above problem, only constraints (IC3), (IC4) and (PC) bind and we also derive some properties of the optimal compensation scheme of the agent.

Proposition 1 *Under Assumptions 1 and 2, there exists a flexible contract implementing action x in θ_1 and y in θ_2. The optimal contract implementing such a profile is obtained as solution of the following simplified problem:*

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \begin{array}{l} p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ \quad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \end{array}$$

$$s.t.\ (IC3),(IC4),(PC)\ \text{holding as equalities and} \quad \bar{w}_1 \geq \bar{w}_2,\ \bar{w}_2 \geq \underline{w}_2$$

and exhibits the following properties:

$$\bar{w}_1 \geq \bar{w}_2 > \underline{w}_2 \geq \underline{w}_1.$$

Recall that (IC3) refers to the "joint deviation" in state θ_1 (i.e., announcing instead the state is θ_2 and choosing action y rather than x), while (IC4) only concerns the misreporting deviation in state θ_2 of announcing θ_1.

Let $u(\theta_1) = -\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} - (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$ denote the agent's expected utility at the optimal contract when state θ_1 occurs; similarly, $u(\theta_2) = -\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}$ is the utility when θ_2 occurs. The properties shown in the above proposition that (IC3) is binding at an optimum and that $\bar{w}_2 > \underline{w}_2$, together with the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$, have the following important implication:

Corollary 1 *At the optimal flexible contract, $u(\theta_2) > u(\theta_1)$.*

Thus even though the less costly action y is implemented in state θ_2 the optimal contract is characterized in that state by a wage that varies with the output realizations. At the same time, the expected utility of the net compensation paid to the manager is higher in state θ_2 than in θ_1. The variability in w_2 and the lack of smoothing in the agent's utility levels across the realizations of θ can both be justified as a way to reduce the variability in the compensation paid in θ_1: it can in fact be verified that (IC3), (IC4) and (PC) can all be satisfied as equality even with a constant level of w_2 - and hence with the same utility levels for the agent in state θ_2 as in θ_1 - but this is suboptimal.

Remark 2 *To further understand the determinants of these properties of the optimal flexible contract, it is useful to compare them with those of the optimal contract (still implementing action x in state θ_1 and y in θ_2) obtained when the realization of θ is publicly* *observable while the action is not. We can show*[6] *that in such case* $\bar{w}_2 = \underline{w}_2$ $\bar{w}_1 > \underline{w}_1$, *and the agent's expected utility is the same in state* θ_1 *as in* θ_2. *Thus the variability in* w_2 *and in the agent's utility levels we found in the optimal flexible contract (Proposition 1) is due to the need of addressing the additional incentive problems arising from the agent's private information over* θ. *A lower variability in* w_2 *could only be achieved, as we already argued, at the cost of a higher variability of* w_1.

3.2 Rigid contracts

The optimal rigid contract implementing a constant action z, $z = x, y$, in every state is obtained as a solution of the following programme (note that the only constraint is given by (PC), no incentive compatibility constraint appears here as the agent has no discretion over the choice of his action):

$$\begin{aligned} \max_{\bar{w}_1, \underline{w}_1, \bar{w}_2, \underline{w}_2} \quad & p[\pi(z,\theta_1)(\bar{R} - \bar{w}_1) + (1-\pi(z,\theta_1))(\underline{R} - \underline{w}_1)] \\ & \quad + (1-p)[\pi(z,\theta_2)(\bar{R} - \bar{w}_2) + (1-\pi(z,\theta_2))(\underline{R} - \underline{w}_2)] \\ (PC) \quad & p[\pi(z,\theta_1)e^{-a(\bar{w}_1 - c_z)} + (1-\pi(z,\theta_1))e^{-a(\underline{w}_1 - c_z)}] + \\ & \quad (1-p)[\pi(z,\theta_2)e^{-a(\bar{w}_2 - c_z)} + (1-\pi(z,\theta_2))e^{-a(\underline{w}_2 - c_z)}] = e^{-a\bar{u}} \end{aligned} \tag{P^{rig}}$$

Its solution is very simple in the present framework: the wage should be constant ($\bar{w}_1 = \underline{w}_1 = \bar{w}_2 = \underline{w}_2 = w_z$), at the level determined by the participation constraint, thus equal to the expected cost of undertaking action z. In particular:

i) Fixed x contract: the compensation is $w_x = \bar{u} + c_x$, and expected profits are:

$$[p\pi(x,\theta_1) + (1-p)\pi(x,\theta_2)]\bar{R} + [p(1-\pi(x,\theta_1)) + (1-p)(1-\pi(x,\theta_2))]\underline{R} - \bar{u} - c_x$$

ii) Fixed y contract: the compensation is $w_y = \bar{u} + c_y$, and profits are:

$$[p\pi(y,\theta_1) + (1-p)\pi(y,\theta_2)]\bar{R} + [p(1-\pi(y,\theta_1)) + (1-p)(1-\pi(y,\theta_2))]\underline{R} - \bar{u} - c_y$$

[6] When θ is observable the only incentive constraints which need to be considered are (IC2), (IC4), the problem is thus clearly simpler and an explicit solution for the optimal compensation scheme can be derived. Interestingly, this turns out to be the same as the optimal contract we obtain when θ is not observable but $\pi(y,\theta_1) > \pi(y,\theta_2)$ (see Appendix B, available online at http://www.eui.eu/Personal/Gottardi/ for a formal derivation).

4 The choice between flexible and rigid contracts

We are now ready to compare the expected profits of the principal at the optimal flexible contract, characterized in Proposition 1, with the expected profits at the rigid contracts specified in the previous section. We can then determine which type of contract is preferable. In particular, we intend to analyze how the superiority of the flexible or of the rigid contractual arrangement depends on various parameters of the environment (the agent's preferences, and in particular his risk attitude, the costs of undertaking the different actions, their probabilities of success, describing both the relative productivity of each action and the relevance of the uncertainty affecting it).

As we said in the previous section, in the optimal flexible contract the agent's action can be adjusted to reflect the different circumstances under which the agent may find himself to operate. However there is also an agency cost in delegating the choice of the action to the agent since the action and the state are not observable and the agent's objectives are not aligned to those of the principal. We should expect therefore that the advantages of flexibility will be higher the bigger is the difference between the relative productivity of the two types of actions in state θ_1 and in the other state θ_2 as well as the smaller is the 'agency cost' which has to be paid to implement the action profile x, y.

4.1 The effect of risk aversion

An important determinant of the agency costs of implementing a variable action profile and hence of the trade-off between flexible and rigid contracts is given by the agent's risk attitude (described, in the case of CARA preferences, by the single parameter a). As shown above, the compensation paid at the rigid contracts is a deterministic amount, independent of the agent's degree of risk aversion. In contrast, at the optimal flexible contract where a variable action profile is implemented, the compensation varies both with θ and the output realizations, and hence the degree of risk aversion matters.

To see the consequences of the agent's risk attitude it is useful to consider first the extreme case where the agent is risk neutral, like the principal. In that case, agency costs are zero as the first best can be implemented, that is the principal can attain the same level of profits as when all incentive compatibility constraints are ignored.

Proposition 2 *When the agent is risk neutral the optimal flexible contract is first best optimal. The expected level of profits is* $p[\pi(x, \theta_1)\bar{R} + (1 - \pi(x, \theta_1))\underline{R}] + (1 - p)[\pi(y, \theta_2)\bar{R} +$

$(1-\pi(y,\theta_2))\underline{R}] - \bar{u} - pc_x - (1-p)c_y$ *and an optimal compensation*[7] *is given by*

$$\begin{aligned}\bar{w}_1 &= \bar{u} + c_x + \frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\ \underline{w}_1 &= \bar{u} + c_x - \frac{\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\ \bar{w}_2 &= \underline{w}_2 = \bar{u} + c_y\end{aligned} \tag{1}$$

Recall that, by Assumption 2(iii), the expected revenue net of the cost is highest with the profile of actions x in state θ_1 and y in θ_2. Since under risk neutrality there are no agency costs and the first best is attainable, it follows that in this case the optimal flexible contract is always preferable to the rigid ones.

On the other hand, when the agent is risk averse ($a > 0$) agency costs are positive, as in order to satisfy the incentive constraints a risk premium must be paid and hence the principal's profits have to be reduced from their first best level. This clearly implies the flexible contract may no longer dominate the rigid contracts.

Note first that Assumption 2(iv) imposes an upper bound on a. For values of a higher than this bound the variable action profile (x, y) is no longer implementable, in which case there is no tradeoff between rigid and flexible contract, and the rigid ones are then always preferable.

Besides the comparison of the extreme values of $a = 0$ and a sufficiently high, where the outcome is clear, we are also interested here in analysing the effects of smaller changes in a, possibly infinitesimal ones, on the relative profitability of flexible and rigid contracts. To this end we need to consider the effects of local changes in risk aversion for the properties of optimal incentive contracts. They prove to be rather complex and no analytic result can be established.[8] Hence in the analysis below we rely on the consideration of a numerical example, for which the optimal payment schedule can be solved numerically. The results obtained prove to be robust to changes in the parameter values chosen.

The parameters describing the environment exhibit the following values:

a	p	$\bar{u}$	$\bar{R}$	$\underline{R}$	c_x	c_y	$\pi(x,\theta_1)$	$\pi(x,\theta_2)$	$\pi(y,\theta_1)$	$\pi(y,\theta_2)$
1	.5	1	10	5	1.5	1	.8	.45	.2	.4

Table 1: Parameter values for the comparative static exercise

[7] Note that this compensation scheme yields $u(\theta_2) = u(\theta_1)$.

[8] It is easy to see that the analysis in Jewitt (1987) is not applicable to the problem under consideration here. The difficulties faced in the comparative statics analysis with respect to risk aversion were also emphasized by Jullien, Salanié and Salanié (1999).

We provide in what follows a characterization of the comparative statics effects of varying the agent's degree of risk aversion a. Figure 1 shows how the difference between the expected profits at the optimal flexible contract and the two rigid contracts changes with a. We see this relationship is monotonically decreasing. For low levels of risk aversion, the flexible contract is preferable to the two rigid contracts, but as a increases the profit differential becomes progressively smaller and eventually, from $a \sim 1.6$ onwards in the situation considered, the rigid contract specifying task x for the agent becomes optimal. This pattern appears to be robust to changes in the value of the other parameters and shows that agency costs are increasing with the agent's risk aversion[9]. Hence we can say that agency costs are increasing and the advantages of delegation decreasing in the agent's degree of risk aversion.

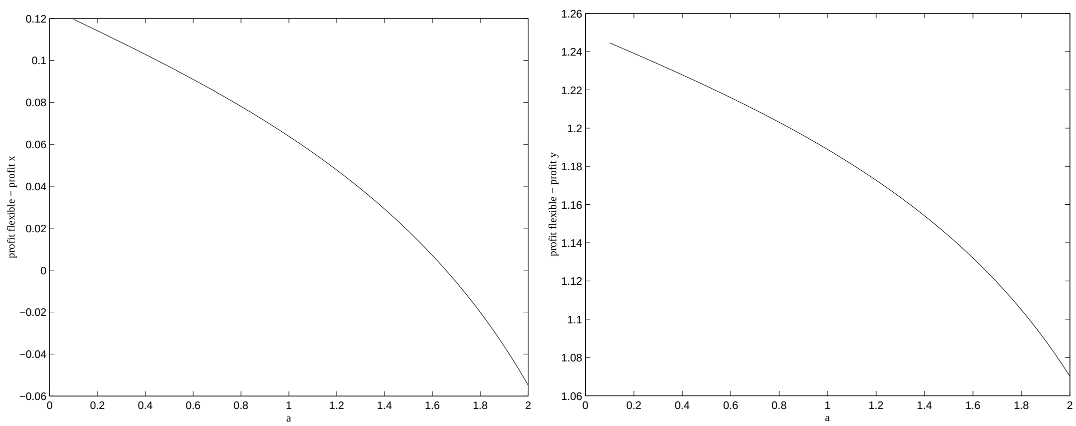

Figure 1: Profit differential between the flexible and rigid contracts as a function of risk aversion

The next two figures illustrate then the implications of the level of the agent's degree of risk aversion for the specific properties of the optimal flexible contract. In particular, Figure 2 describes the effect of varying a on the spread between the compensation paid for the high and low output realizations at the optimal flexible contract respectively in state θ_1 (i.e. $\bar{w}_1 - \underline{w}_1$) and θ_2. It shows that the spread in state θ_1 is first decreasing and then increasing in a while the spread in θ_2 is always increasing in a. Figure 3 shows that the utility differential also varies non monotonically with a, first increasing and then decreasing. We should point out however that the properties found in Figures 2 and 3,

[9] A similar pattern also obtains when the realization of θ is commonly observed: increasing risk aversion makes the rigid contracts more attractive relative to the flexible ones. The profits of the flexible contract when θ is observable are strictly higher than when θ is only privately observed, and we find the difference is increasing in risk aversion.

unlike those of Figure 1, are not quite robust to changes in the values of the parameters considered in Table 1[10].

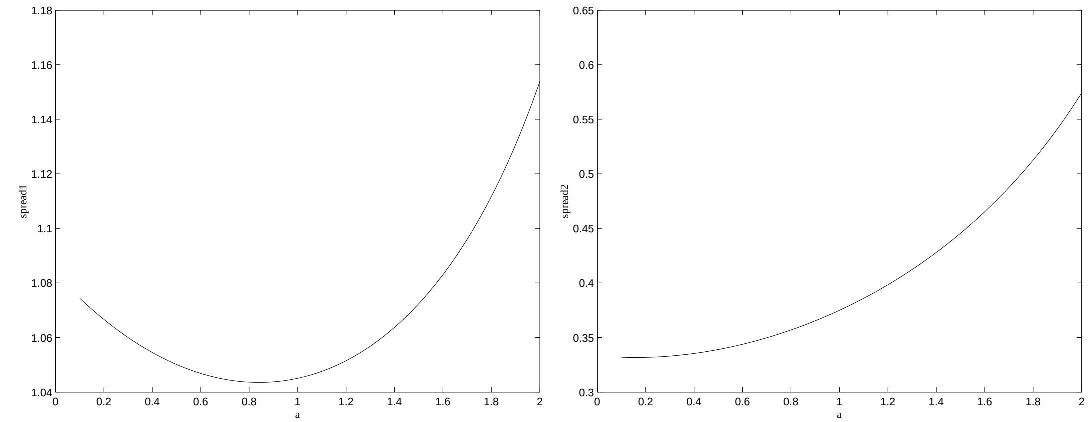

Figure 2: Wage differentials at the optimal flexible contract as a function of risk aversion

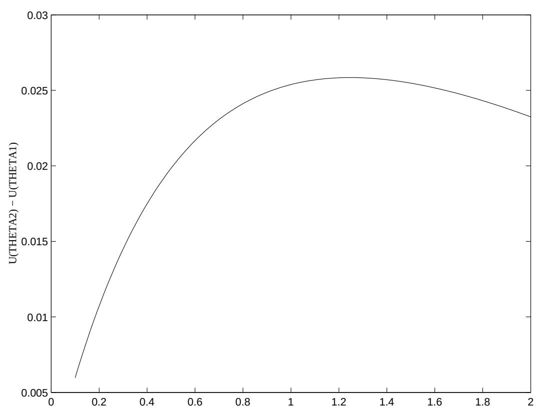

Figure 3: Utility differential $u(\theta_2) - u(\theta_1)$ at the optimal flexible contract as a function of a

Trying to disentangle the various effects of risk aversion, we can first observe that increasing a makes the participation constraint, ceteris paribus, harder to satisfy: such constraint requires that the certainty equivalent of the lottery with outcomes $\bar{w}_1 - c_x, \underline{w}_1 - c_x$, $\bar{w}_2 - c_y, \underline{w}_2 - c_y$ is equal to $\bar{u}$, but the certainty equivalent of this lottery decreases with risk aversion.

[10]Even when θ is observable we find for instance that the spread of the compensation paid in state θ_1 can be non monotonic or monotonically decreasing depending on the values of the parameters.

Each of the two incentive constraints which are binding at an optimum solution, (IC3) and (IC4), then requires a pair of distinct lotteries to have the same expected utility. In the case of (IC3) we cannot rank the two lotteries that are compared in terms of riskiness, $(\bar{w}_1 - c_x, \underline{w}_1 - c_x)$, with probabilities $\pi(x, \theta_1), 1 - \pi(x, \theta_1)$, and $(\bar{w}_2 - c_y, \underline{w}_2 - c_y)$, with probabilities $\pi(y, \theta_1), 1 - \pi(y, \theta_1))$. We know in fact that $\underline{w}_1 - c_x$ is the smallest outcome but we do not know, for instance, how to rank $\bar{w}_1 - c_x$ versus $\bar{w}_2 - c_y$. Furthermore, the attached probabilities are not the same. Thus, the effect of changing risk aversion on this constraint is ambiguous. On the other hand, for (IC4) we can say that the second of the two lotteries compared is always riskier than the first one. Hence increasing risk aversion loosens this constraint: i.e., if a is increased while the compensation is kept constant, the constraint becomes slack. Hence, when a increases, (PC) is harder to satisfy while (IC4) is easier, and the effect on (IC3) is unclear.

4.2 The effect of actions' productivity and cost

We investigate next how the relative profitability of flexible versus rigid contracts is affected by the following parameters: the levels of the probability of success for each action and event in which it is undertaken and the cost of the different types of actions c_z. Our findings, still based on the parametrization described in Table 1, are summarized in Table 2. A + (resp. -) sign indicates that an increase in the parameter value indicated in the top of the column always increases (decreases) the variable appearing in the row, while a ? indicates the effect is ambiguous, not always of the same sign.

Parameter Range	$\pi(x, \theta_1)$ [.75,.9]	$\pi(x, \theta_2)$ [.35,.55]	$\pi(y, \theta_1)$ [.15,.25]	$\pi(y, \theta_2)$ [.3,.5]	c_x [1.2,1.8]	c_y [.7,1.25]
Profit flexible - profit x	$+$	$-$	$-$	$+$	?	?
Profit flexible - profit y	$+$	$=$	$-$	$-$	$-$	$+$
$\bar{w}_1 - \underline{w}_1$	$-$	$=$	$+$	?	$+$	$-$
$\bar{w}_2 - \underline{w}_2$	$-$	$=$	$-$	$+$	$+$	$-$
$u(\theta_2) - u(\theta_1)$	$-$	$=$	$-$	$+$	$+$	$-$

Table 2: Comparative statics with respect to probabilities and costs

For instance, the first column reports the sign of the effects of increasing $\pi(x, \theta_1)$, within the interval indicated, [.75, .9] on the following variables: (i) the differential between the expected profits at the optimal flexible contract and those at the x rigid contract in the first row and at the y one in the second row; (ii) the spread between the compensation paid for the high and low realization of the output when state θ_1 occurs in the third row and when θ_2 occurs in the fourth one; (iii) the difference in expected utility in the two states. All this when the other parameters are kept fixed at the values indicated in Table 1.

In particular, we find that the profitability of the flexible contract, relative to both rigid contracts, increases if $\pi(x, \theta_1)$ (probability of success with action x in state 1) increases, or $\pi(y, \theta_1)$ decreases. Such changes increase the productivity of the costlier action (x) relative to the less costly one in state θ_1 as well as (in the first case) relative to state θ_2. The same effects are obtained with a decrease in $\pi(x, \theta_2)$, reducing the difference between the productivity of actions x and y in state θ_2. On the other hand, a change in $\pi(y, \theta_2)$ has opposite effects on the profitability of the flexible contract relative to the two rigid ones, while the effect of increasing the costs c_x and c_y of the two actions on the same profit difference is non monotonic.

We also see that the variability in the compensation paid in state θ_2, where the less costly action is implemented, always moves in the same direction as the utility differential $u(\theta_2)-u(\theta_1)$, suggesting these two are complementary instruments to address the incentive problems generated by the private information over θ, as already mentioned in Remark 2.

5 The choice of delegation with ambiguity

We examine now the case where, at the time in which the contract is written, the information available to the parties concerning the likelihood of the various events is not precise enough for them to have a sharp probability belief. This appears rather natural in many instances, where the situation faced by the parties is sufficiently new that past data cannot be used to pin down probabilities.

We thus assume in this section that there is a set of probability distributions over $\{\theta_1, \theta_2\}$, which is described by an interval of values for the probability of θ_1 occurring, $p \in [\underline{p}, \bar{p}]$. This set represents the probability beliefs consistent with the available information (precise information corresponds to a singleton set, $\underline{p} = \bar{p}$, so there is only one probability distribution compatible with the available information). Similarly, there is a set of possible probabilities of $\bar{R}$ occurring, conditionally on action x and state θ_1 given by the interval $[\underline{\pi}(x, \theta_1), \bar{\pi}(x, \theta_1)]$, another set for the beliefs conditionally on action y and state θ_2, given by the interval $[\underline{\pi}(y, \theta_2), \bar{\pi}(y, \theta_2)]$, conditionally on action y and state θ_1 given by the set $[\underline{\pi}(y, \theta_1), \bar{\pi}(y, \theta_1)]$ and conditionally on action x in state θ_2, given by the set $[\underline{\pi}(x, \theta_2), \bar{\pi}(x, \theta_2)]$.

We need a tractable model of decision under uncertainty in such situations, that allows for a simple parametrization of individuals' attitude towards uncertainty, and in particular of their ambiguity (or imprecision as we will call it) aversion. We use the model developed by Gajdos, Hayashi, Tallon, Vergnaud (2008), to which we refer the reader for further details. In the case of interest here, this model is particularly simple. The criterion consists in taking a convex combination of the minimal expected utility (with respect to all possible distributions in the specified intervals) and the expected utility with respect to a central probability (the center of the interval). The weight α placed on the minimal expected utility in this combination reflects the decision maker's imprecision aversion. The case $\alpha = 0$ reflects imprecision neutrality: the decision maker acts as if he were an expected utility maximizer with respect to the central probability in each interval, while $\alpha = 1$ reflects extreme imprecision aversion, the decision maker putting all the weight on the least favorable prior.[11]

In this section we assume that both parties are risk neutral. We furthermore assume that the Principal is imprecision neutral. He therefore acts as an expected profit maximizer, with respect to the central probability. The Agent, on the other hand, is characterized by his degree α of imprecision aversion. Variables with $\hat{}$ represent the "central probabilities", i.e., $\hat{p} = \frac{\underline{p}+\bar{p}}{2}$, $\hat{\pi}(x,\theta_1) = \frac{\underline{\pi}(x,\theta_1)+\bar{\pi}(x,\theta_1)}{2}$ and so on.

The Principal's objective function – when implementing the flexible contract – is then simply to maximize

$$\hat{p}[\hat{\pi}(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\hat{\pi}(x,\theta_1))(\underline{R}-\underline{w}_1)]+(1-\hat{p})[\hat{\pi}(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\hat{\pi}(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

Let's now consider the Agent's incentive constraints. His utility in state θ_1 when action x is exerted is given by

$$\alpha \min_{\pi(x,\theta_1)\in[\underline{\pi}(x,\theta_1),\bar{\pi}(x,\theta_1)]} \{\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1\} + (1-\alpha)\left[\hat{\pi}(x,\theta_1)\bar{w}_1 + (1-\hat{\pi}(x,\theta_1))\underline{w}_1\right]$$

This can also be expressed as

$$\min_{\pi\in[\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1),\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1)]} \{\pi\bar{w}_1 + (1-\pi)\underline{w}_1\},$$

where $\alpha(x,\theta_1) = \alpha\frac{\bar{\pi}(x,\theta_1)-\underline{\pi}(x,\theta_1)}{2}$.

[11] This criterion, as we see clarly from the expression of the agents' utility functions in what follows, belongs to the general class of multiple priors models due to Gilboa and Schmeidler (1989). One important difference is that, here, information is made explicit, in the form of sets of probability distributions. This allows one to define a measure of imprecision aversion, something that is not possible in the original model of Gilboa and Schmeidler (1989).

To simplify notation, denote $I(x,\theta_1)$ the interval $[\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1), \hat{\pi}(x,\theta_1)+\alpha(x,\theta_1)]$. Using similar notation for action y in state θ_2, and action y in state θ_1, the incentive constraints, analogous to those in (P^{flex}), take the following expressions:

$$\begin{cases} (IC1^*) \ \min_{\pi\in I(x,\theta_1)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1] & \geq \ \min_{\pi\in I(x,\theta_1)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2] \\ (IC2^*) \ \min_{\pi\in I(x,\theta_1)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1]-c_x & \geq \ \min_{\pi\in I(y,\theta_1)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1]-c_y \\ (IC3^*) \ \min_{\pi\in I(x,\theta_1)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1]-c_x & \geq \ \min_{\pi\in I(y,\theta_1)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2]-c_y \\ & \\ (IC4^*) \ \min_{\pi\in I(y,\theta_2)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2] & \geq \ \min_{\pi\in I(y,\theta_2)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1] \\ (IC5^*) \ \min_{\pi\in I(y,\theta_2)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2]-c_y & \geq \ \min_{\pi\in I(x,\theta_2)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2]-c_x \\ (IC6^*) \ \min_{\pi\in I(y,\theta_2)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2]-c_y & \geq \ \min_{\pi\in I(x,\theta_2)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1]-c_x \end{cases} \tag{2}$$

The participation constraint takes then the following form (PC^*):

$$\min_{p\in[\hat{p}-\alpha(p),\hat{p}+\alpha(p)]}\left\{p\min_{\pi\in I(x,\theta_1)}[\pi\bar{w}_1+(1-\pi)\underline{w}_1]-c_x+(1-p)\min_{\pi\in I(y,\theta_2)}[\pi\bar{w}_2+(1-\pi)\underline{w}_2]-c_y]\right\}\geq\bar{u}$$

where $\alpha(p)=\alpha\frac{\bar{p}-\underline{p}}{2}$.

Finally, it is convenient, in order to make a comparison with the previous analysis, to reformulate Assumption 2(ii) in the present framework as follows:

$$\text{for all } \pi(x,\theta_1)\in I(x,\theta_1), \pi(x,\theta_2)\in I(x,\theta_2), \pi(y,\theta_2)\in I(y,\theta_2), \pi(y,\theta_1)\in I(y,\theta_1),$$
$$\text{we have } \pi(x,\theta_1)>\pi(x,\theta_2)>\pi(y,\theta_2)>\pi(y,\theta_1)$$

This ensures that there is no overlap in the (induced) probability intervals, and thus that the induced beliefs (no matter what they are) respect the ordering we imposed in the previous sections when these beliefs were assumed to be precise, single probability distributions. Note the one above is a joint assumption on α, the imprecision aversion of the Agent, and the "amount of imprecision", captured by the width of the probability intervals.

5.1 Existence of an agency cost

Let us start from the full insurance contract, that is the optimal contract absent any informational asymmetries. This contract insures the worker within each state θ as well as across states. It has $\bar{w}_1=\underline{w}_1=\bar{u}+c_x$ and $\bar{w}_2=\underline{w}_2=\bar{u}+c_y$. We show now that any deviation from this contract that satisfies the participation constraint, (weakly) decreases the Principal's profits.

For instance, perturb the full insurance contract in the following way: $d\bar{w}_1>0$, $d\underline{w}_1<0$, $d\bar{w}_2>0$ and $d\underline{w}_2<0$ so that

$$(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))d\bar{w}_2+(1-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2))d\underline{w}_2>(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))d\bar{w}_1+(1-\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1))d\underline{w}_1$$

i.e., the Agent is now better off in state θ_2 than in θ_1. As a consequence, the Agent now evaluates the occurrence of θ_1 with the least favorable distribution, i.e., $\hat{p}+\alpha(p)$. Similarly, he uses $\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)$ to evaluate, within state θ_1, the probability of $\bar{R}$, conditionally on doing action x, and $\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)$ to evaluate, within state θ_2 the probability of $\bar{R}$, conditionally on doing action y.

The change in expected cost for the Principal is equal to

$$\hat{p}[\hat{\pi}(x,\theta_1)d\bar{w}_1+(1-\hat{\pi}(x,\theta_1))d\underline{w}_1]+(\hat{\pi}(y,\theta_2)d\bar{w}_2+(1-\hat{\pi}(y,\theta_2))d\underline{w}_2,$$

and we show it is non negative if the participation constraint has to be satisfied. The effect of this change in compensation on the Agent's participation constraint is in fact

$$\begin{gathered}(\hat{p}+\alpha(p))\left\{(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))d\bar{w}_1+(1-\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1))d\underline{w}_1\right\}+\\(1-\hat{p}-\alpha(p))\left\{(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))d\bar{w}_2+(1-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2))d\underline{w}_2\right\}\geq 0\end{gathered}$$

which can be decomposed as follows:

$$\begin{aligned}&\hat{p}[\hat{\pi}(x,\theta_1)d\bar{w}_1+(1-\hat{\pi}(x,\theta_1))d\underline{w}_1]+(1-\hat{p})[\hat{\pi}(y,\theta_2)d\bar{w}_2+(1-\hat{\pi}(y,\theta_2))d\underline{w}_2]+\\&\hat{p}\alpha(x,\theta_1)(d\underline{w}_1-d\bar{w}_1)+(1-\hat{p})\alpha(y,\theta_2)(d\underline{w}_2-d\bar{w}_2)+\\&\alpha(p)[(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))d\bar{w}_1+(1-\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1))d\underline{w}_1\\&\qquad-(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))d\bar{w}_2-(1-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2))d\underline{w}_2]\geq 0\end{aligned}\tag{3}$$

The term in the first line is equal to the change in cost for the Principal. The term in the second line is negative, given the sign of the deviations. The last term is negative since the deviation considered implies that the utility in state θ_2 is higher than in state θ_1. Hence, for the participation constraint to still hold ((3) to be satisfied) it has to be the case that costs increase for the Principal and profits decrease.[12]

The deviation contemplated above generates a higher utility level in state θ_2 than in θ_1 as well as, in each θ state, for the high income realization. This pins down the induced beliefs that appear in the Agent's participation constraint. The same type of reasoning can be applied for any other deviation from the full insurance contract – with different induced beliefs – to show that expected costs increase. Since we know that a constant level of wages in state θ_1 violates incentive compatibility, we conclude therefore that there exists an agency cost[13]. Thus the sole presence of imprecision aversion generates an agency cost, analogously to risk aversion.

[12]This is true except in the special case where $\alpha(x,\theta_1)=\alpha(y,\theta_2)=\alpha(p)=0$, which occurs for instance if the Agent is imprecision neutral. In this case the terms appearing in the second to the fourth line in (3) are all zero. Hence (3) is exactly equal to the change in cost for the Principal, that is the considered change in wages has no effect on profits, and so the first best level of welfare is still attainable.

[13]This cost could be zero, as we have seen in footnote 12, in some special cases.

What this analysis also shows is that, in line with the no trade results present in the literature on ambiguity aversion (see, e.g., Billot et al. (2000), Strzalecki and Werner (2011)) it is never optimal to induce different beliefs between the Agent and the Principal, unless it is required to do so in order to satisfy the incentive constraints. And in this setting such difference in beliefs never increases the surplus to be split between the two parties.

5.2 Optimal contract

We provide here a partial characterization of the optimal flexible contract under imprecision aversion and analyze then the relative profitability of flexible vs. rigid contracts.

Proposition 3 *For an open set of values of parameters describing the environment, the optimal flexible contract is given by*

$$\begin{aligned} \bar{w}_1 &= \bar{u} + c_x + \frac{1-\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)}{\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)}\Delta c \\ \underline{w}_1 &= \bar{u} + c_x - \frac{\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)}{\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)}\Delta c \\ \bar{w}_2 &= \underline{w}_2 = \bar{u} + c_y \end{aligned} \tag{4}$$

Otherwise, the optimal flexible contract is characterized by $\bar{w}_2 > \underline{w}_2$ and by a lower variability of wages in state θ_1.

The contract described in the above proposition is the one which we saw in Proposition 2 allows to attain the first best in the risk neutral, imprecision neutral case. Observe that, at such contract, the incentive constraints $(IC3^*)$ and $(IC4^*)$ in (2) are binding, while the others are slack. The participation constraint (PC^*) is also binding.

We outline here the argument to establish the result in Proposition 3, referring to the Appendix for further details. We investigate whether local deviations from this contract $(d\bar{w}_1, d\underline{w}_1, d\bar{w}_2, d\underline{w}_2)$, satisfying $(IC3^*)$, $(IC4^*)$ and (PC^*) as equality, can increase the expected level of the Principal's profit. To that end, we use the system given by the three binding constraints to solve for $d\bar{w}_1$, $d\bar{w}_2$, and $d\underline{w}_2$ as a function of $d\underline{w}_1$. When $d\underline{w}_1$ is positive, we have $d\bar{w}_1 < 0$, $d\bar{w}_2 > 0$, and $d\underline{w}_2 < 0$; the opposite signs when $d\underline{w}_1$ is negative.

Plug then the expressions obtained as in the previous paragraph for $d\bar{w}_1$, $d\bar{w}_2$, and $d\underline{w}_2$ as a function of $d\underline{w}_1$, together with $d\underline{w}_1$, in the derivative of the profit function of the Principal. It is immediate to verify that this derivative is always negative for the deviation characterized by $d\underline{w}_1 < 0$, which is so not profitable for the Principal.

On the other hand, whether the deviation with $d\underline{w}_1 > 0$ is profitable or not depends on the parameter values of the model. We can show, for instance, that when $\alpha(p) =$

$\alpha(y,\theta_2)=0$ while $\alpha(x,\theta_1)>0$ this deviation is profitable and so the considered contract is not optimal. On the other hand, when $\alpha(x,\theta_1)=0$ this deviation too is not profitable and the proposed contract is then optimal. Both properties are actually true for an open set of parameters around the points indicated.

Recall that $\alpha(x,\theta_1)=\alpha\frac{\bar{\pi}(x,\theta_1)-\underline{\pi}(x,\theta_1)}{2}$. This term encapsulates both imprecision (as measured by the width of the interval $[\underline{\pi}(x,\theta_1),\bar{\pi}(x,\theta_1)]$) and imprecision aversion (as captured by the parameter α). Thus, the case $\alpha(p)=\alpha(y,\theta_2)=0$ and $\alpha(x,\theta_1)>0$ corresponds to a situation where there is no imprecision on p, i.e., there is a known probability of occurrence of θ_1 or θ_2, no imprecision regarding the probability of success in state θ_2 when y is undertaken, and on the contrary there is imprecision on the probability of success in state θ_1. In this case a reduction in the wage volatility in state θ_1 (as in the considered deviation with $d\underline{w}_1>0$) allows to increase profits, even though the wage volatility in state θ_2 increases. Clearly it is not possible to bring down to zero the volatility of wages in θ_1, as some variability in this state is still required to implement action x. We conjecture that the optimal contract in this case is obtained at the point where $(IC2^*)$ binds.

In contrast, when $\alpha(x,\theta_1)=0$ there is no imprecision on the probability of success in state θ_1 when x is undertaken. In this situation, it is actually possible to attain the first best level of profits. The three terms appearing in the second to the fourth line of (3) that we identified as constituting the agency cost, are in fact all zero, as there is no variability in the utility across states, nor in w_2, the only variability is in w_1 and $\alpha(x,\theta_1)=0$.

When we consider the case in which all intervals considered have equal imprecision, that is

$$\bar{\pi}(x,\theta_1)-\underline{\pi}(x,\theta_1)=\bar{\pi}(y,\theta_2)-\underline{\pi}(y,\theta_2)=\bar{p}-\underline{p}\equiv\beta$$

and thus $\alpha(p)=\alpha(y,\theta_2)=\alpha(x,\theta_1)\equiv\alpha\beta$, we get the following expression for the change in the Principal's profit:

$$\alpha\beta\frac{(\hat{\pi}(y,\theta_2)-\hat{\pi}(y,\theta_1))(-1+\hat{\pi}(x,\theta_1)-\hat{\pi}(y,\theta_2))+(1-\hat{p})(\hat{\pi}(x,\theta_1)-\hat{\pi}(y,\theta_1))}{[\hat{\pi}(y,\theta_1)-\hat{\pi}(y,\theta_2)][(\hat{p}+\alpha\beta)(\hat{\pi}(x,\theta_1)-\alpha\beta)+(1-\hat{p}-\alpha\beta)(\hat{\pi}(y,\theta_2)-\alpha\beta)]}$$

It is easy to show that again this expression is positive for an open set of parameters.

The conclusion we draw from this analysis is that the optimal flexible contract under imprecision aversion (and risk neutrality) is different from the optimal one under risk aversion (and imprecision neutrality). In the latter it is never optimal to provide full insurance to the Agent in state θ_2 as well as across states θ_1 and θ_2, while this is optimal, for an open set of parameter values, under imprecision aversion. When this configuration is suboptimal also with imprecision aversion, the optimal flexible contract exhibits less volatility in state θ_1, together with some volatility in state θ_2 and across the θ states, as when there is risk aversion.

Remark 3 *As we have seen, imprecision aversion acts as if "inducing" different beliefs between the Principal and the Agent. The induced beliefs depend on the feature of the payment scheme. To better understand the role played by this heterogeneity in beliefs, it is useful to examine the case where agents' beliefs are fixed at this induced level. Consider in particular the contract described in (4): the Principal uses beliefs* $\hat{\pi}(x, \theta_1)$ *while the Agent uses* $\bar{\pi}(x, \theta_1)$. *On the other hand, the beliefs of the Agent over* θ_1 *and on* $\bar{R}$ *conditionally on being in state* θ_2 *and on doing action* y *are not pinned down and could be set equal to those used by Principal,* $\hat{p}$ *and* $\hat{\pi}(y, \theta_2)$. *However when the Agent and the Principal have exogenously fixed beliefs set at this level (*$\bar{\pi}(x, \theta_1), \hat{p}$, $\hat{\pi}(y, \theta_2)$ *for the Agent and* $\hat{\pi}(x, \theta_1), \hat{p}$, $\hat{\pi}(y, \theta_2)$ *for the Principal), the contract considered is never optimal: a higher level of expected profits can in fact be attained by reducing the volatility of the payment in the* θ_1 *state and increasing that in* θ_2 *and across the* θ *states. This stands in stark contrast to the imprecision aversion case, where the contract described is optimal for an open set of parameter values. The reason is precisely because the above deviation would induce a change in the Agent's beliefs which would make the deviation no longer profitable.*

We can then compare the optimal flexible contract to the rigid contracts. In particular we analyze how the relative profitability of the two varies in this case with respect to the parameters describing the imprecision (that is, the intervals of possible probability levels) and the imprecision aversion (α).

Corollary 2 *When the optimal flexible contract is the one in (4), expected profits are decreasing in* α.

For the open set of parameter values for which the contract described in (4) is the optimal flexible contract, we can easily see that the effect on the Principal's profit of increasing the Agent's imprecision aversion is unambiguously negative. The expected wage bill the Principal has to pay is in fact in this case given by

$$\hat{p}(\hat{u} + c_x) + (1 - \hat{p})(\bar{u} + c_y) + \hat{p}\frac{\alpha(x, \theta_1)}{\hat{\pi}(x, \theta_1) - \hat{\pi}(y, \theta_2) - (\alpha(x, \theta_1) - \alpha(y, \theta_2))}\Delta c$$

Recall that $\alpha(x, \theta_1) = \alpha\frac{\bar{\pi}(x,\theta_1) - \underline{\pi}(x,\theta_1)}{2}$ and $\alpha(y, \theta_2) = \alpha\frac{\bar{\pi}(y,\theta_2) - \underline{\pi}(y,\theta_2)}{2}$ and substitute these terms in the above expression. If we then differentiate it with respect to α we readily see that the expected wage bill is always increasing in α. Increasing the degree of imprecision aversion will therefore lower expected profits at the flexible contract and favor rigid contracts, whose profits are independent of α. The same is clearly true for increases in the Agent's imprecision, that is of the width of the interval $[\bar{\pi}(x,\theta_1) - \underline{\pi}(x,\theta_1)]$.

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Appendix

Proof of Proposition 1

The proof is decomposed into three Propositions (A.1 to A.3)

Proposition A.1: *At an optimal flexible contract the compensation exhibits the following properties:* $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$, *and* $\bar{w}_1 > \underline{w}_1$. *Furthermore:*
(i) if $\underline{w}_2 > \underline{w}_1$, *then* $\bar{w}_1 > \bar{w}_2$ *and (IC3) and (IC4) are binding, while (IC1), (IC2), (IC5) and (IC6) are slack.*
(ii) if $\underline{w}_2 = \underline{w}_1$, *then* $\bar{w}_1 = \bar{w}_2$ *and (IC3) binds, while (IC1), (IC2), and (IC4) are automatically satisfied ((IC1) and (IC4) as equalities), and (IC5) and (IC6) are slack.*[14]

Proof.

Step 1: At an optimal solution $\bar{w}_2 \geq \underline{w}_2$.

Proof. Suppose not, that is, $\bar{w}_2 < \underline{w}_2$.

Then, it is immediate to show, given that $c_y < c_x$, $\pi(y, \theta_1) < \pi(x, \theta_1)$, and $\pi(y, \theta_2) < \pi(x, \theta_2)$, that both (IC1) and (IC5) are slack. Start with (IC1): the right hand side of (IC1) is strictly greater than the right hand side of (IC3) and hence, (IC1) is slack. For (IC5), rewrite the constraint as:

$$[\pi(y,\theta_2)e^{-a\bar{w}_2} + (1-\pi(y,\theta_2))e^{-a\underline{w}_2}]e^{ac_y} \leq [\pi(x,\theta_2)e^{-a\bar{w}_2} + (1-\pi(x,\theta_2))e^{-a\underline{w}_2}]e^{ac_x}$$

Then, under the assumption, the expression in bracket in the left hand side is strictly smaller than the one in the right hand side, which implies, together with the order on the cost, that (IC5) is slack.

We now show that if $\bar{w}_2 < \underline{w}_2$, then it is possible to find an improvement for the principal by pushing $\bar{w}_2$ and $\underline{w}_2$ closer. Consider $\Delta\bar{w}_2 > 0$ and $\Delta\underline{w}_2 < 0$ (i.e. a discrete change in $\bar{w}_2, \underline{w}_2$) such that:

(i) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}$ and,

(ii) $\pi(y,\theta_2)\Delta\bar{w}_2 + (1-\pi(y,\theta_2))\Delta\underline{w}_2 < 0$

Note that it is possible to find such a $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$ by concavity of the utility function. By condition (ii), we can conclude that this change improves the principal's profit. It remains to show that it is feasible and satisfies the remaining incentive and the participation constraints.

[14]The argument shows that the stated result holds whenever the agent's utility function can be decomposed as $u(w-c) = u(w)u(-c)$ with u (strictly) concave and increasing (i.e. not only for , CARA).

(IC2) is trivially satisfied since it does not depend on $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$. (IC4) and (IC6) are satisfied by construction, given condition (i) and the same is true for (PC). Thus, it remains to show that (IC3) holds. Given that the left hand side of (IC3) remains unchanged, it is enough to show that:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)}$$

This follows from condition (i) and the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$. Indeed, (i) is equivalent to $\pi(y,\theta_2)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_2))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] = 0$. The first term is negative while the second is positive, so we have, given that $\pi(y,\theta_1) < \pi(y,\theta_2)$,

$\pi(y,\theta_1)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_1))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] > 0$, which yields the desired result. □

Step 2: At an optimal solution $\bar{w}_1 > \underline{w}_1$.

Proof. This is a direct consequence of (IC2). □

Step 3: At an optimal solution (IC3) binds.

Proof. We distinguish two cases, according to whether $\underline{w}_2 = \bar{w}_2$ or $\underline{w}_2 < \bar{w}_2$.

Case 1.: $\underline{w}_2 = \bar{w}_2 \equiv w_2$.

In that event, (IC5) is automatically satisfied and therefore can be dropped. Furthermore, (IC3) implies (IC1) which can so also be dropped. Now, by Step 2 $\underline{w}_1 < \bar{w}_1$. Hence, given that $\pi(y,\theta_2) > \pi(y,\theta_1)$, it is possible to show that (IC3) and (IC4) imply (IC6), which can be dropped.

Obviously, (IC3) and (IC6) cannot be simultaneously binding. We show next that (IC3) has to bind and therefore (IC6) is slack. Assume not, i.e., (IC3) is slack and consider (an infinitesimal change) $d\bar{w}_1 < 0$, $d\underline{w}_1 = 0$ and $dw_2 > 0$. Since (IC3) is slack, for sufficiently small such quantities it continues to hold. (IC4) and (IC6) remain satisfied. Choosing $d\bar{w}_1 = -\frac{(1-p)e^{-a(w_2-c_y)}}{p\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}}dw_2$ ensures that the participation constraint continues to hold. By construction, the change in the objective function is equal to $(1-p)\left[\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}} - 1\right]dw_2$. Given that $dw_2 > 0$, this quantity is positive (hence leading to an increase in the objective function) if $e^{-a(w_2-c_y)} > e^{-a(\bar{w}_1-c_x)}$, that is if $\bar{w}_1 > w_2 + \Delta c$. This property always holds in the case under consideration ($\underline{w}_2 = \bar{w}_2$): (IC3) can in fact be rewritten as follows:

$$\pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1} \leq e^{-a(w_2+\Delta c)},$$

which in turn implies, together with the property $\bar{w}_1 > \underline{w}_1$ established in Step 2, that $e^{-a\bar{w}_1} < e^{-a(w_2+\Delta c)}$, and therefore $\bar{w}_1 > w_2 + \Delta c$.

Hence, whenever (IC3) is slack we can find a perturbation of the wage bill that increases the Principal's profit, contradicting optimality of the contract. Therefore (IC3) has to bind (and hence (IC6) is slack).

Case 2.: $\underline{w}_2 < \bar{w}_2$.

Assume (IC3) is slack and consider a discrete change $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ such that: (i) $\pi(y,\theta_2)\Delta\bar{w}_2 + (1-\pi(y,\theta_2)\Delta\underline{w}_2 < 0$ and (ii) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2)} + (1-\pi(y,\theta_2))e^{-a(\bar{w}_2+\Delta\underline{w}_2)} = \pi(y,\theta_2)e^{-a\bar{w}_2} + (1-\pi(y,\theta_2))e^{-a\bar{w}_2}$. Such numbers exist by strict concavity of u.

Notice that (IC2), (IC4), (IC6) and (PC) are unaffected by these changes and thus continue to hold. We now check (IC1). The left hand side is unchanged and we therefore need to show that: $\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_x)}$, which is equivalent to

$$\pi(x,\theta_1)[e^{-a\bar{w}_2} - e^{-a(\bar{w}_2+\Delta\bar{w}_2)}] + (1-\pi(x,\theta_1))[e^{-a\underline{w}_2} - e^{-a(\underline{w}_2+\Delta\underline{w}_2)}] \leq 0$$

But this holds as a consequence of (ii), given that $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ and $\pi(x,\theta_1) > \pi(y,\theta_2)$. Thus, (IC1) continues to hold.

It remains to check (IC5). By construction, the left hand side is unaffected by the change. Given that $\pi(x,\theta_2) > \pi(y,\theta_2)$, one can replicate the argument showing that (IC1) holds to prove that (IC5) holds as well. $\square$

Step 4: At an optimal solution (IC6) is slack.

Proof. Given that $\bar{w}_2 \geq \underline{w}_2$ and $\pi(y,\theta_2) \geq \pi(y,\theta_1)$, we have

$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$.

From the previous step, we know (IC3) is binding, and hence

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$$

Given that $\bar{w}_1 > \underline{w}_1$ and $\pi(x,\theta_1) \geq \pi(x,\theta_2)$, this establishes that (IC6) is slack, i.e.

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} < \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}$$

$\square$

Step 5: At an optimal solution (IC5) is slack.

Proof. If $\bar{w}_2 = \underline{w}_2$, this is obvious. Consider next the case $\bar{w}_2 > \underline{w}_2$. Then, $\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$. From Step 3 we know that (IC2) binds, i.e., $\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} = \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$.

Now, by (IC1), $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}\leq\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)}$ and hence, since $\bar{w}_2>\underline{w}_2$ and $\pi(x,\theta_1)>\pi(x,\theta_2)$, $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}<\pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$. As a consequence,

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}<\pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$$

showing that (IC5) is slack. $\square$

Step 6: At an optimal solution, $\bar{w}_1\geq\bar{w}_2$ and $\underline{w}_1\leq\underline{w}_2$. Furthermore, if $\underline{w}_1=\underline{w}_2$, then it must be the case that $\bar{w}_1=\bar{w}_2$.

Proof. Rewrite (IC1) and (IC4) as follows:

$$\begin{aligned}\pi(x,\theta_1)\left[e^{-a\bar{w}_1}-e^{-a\bar{w}_2}\right] &\leq (1-\pi(x,\theta_1))\left[e^{-a\underline{w}_2}-e^{-a\underline{w}_1}\right] &(5)\\ \pi(y,\theta_2)\left[e^{-a\bar{w}_2}-e^{-a\bar{w}_1}\right] &\leq (1-\pi(y,\theta_2))\left[e^{-a\underline{w}_1}-e^{-a\underline{w}_2}\right] &(6)\end{aligned}$$

Assume $\bar{w}_1<\bar{w}_2$, then (5) implies that $\underline{w}_1>\underline{w}_2$ and (5) and (6) yield that:

$$\frac{\pi(x,\theta_1)}{1-\pi(x,\theta_1)}\leq\frac{e^{-a\underline{w}_2}-e^{-a\underline{w}_1}}{e^{-a\bar{w}_1}-e^{-a\bar{w}_2}}\leq\frac{\pi(y,\theta_2)}{1-\pi(y,\theta_2)}$$

But this is not possible given that $\pi(y,\theta_2)<\pi(x,\theta_1)$. Hence, $\bar{w}_1\geq\bar{w}_2$. A similar argument establishes that $\underline{w}_1\leq\underline{w}_2$.

Finally, suppose that $\underline{w}_1=\underline{w}_2$. Then, using the fact that (IC3) is binding, one can rewrite (IC2) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}\leq\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$$

which yields $\bar{w}_1=\bar{w}_2$, since we assumed that $\underline{w}_1=\underline{w}_2$ and we proved above that $\bar{w}_1\geq\bar{w}_2$. . $\square$

Step 7: At an optimal solution (IC2) is slack if $\underline{w}_1<\underline{w}_2$. If $\underline{w}_1=\underline{w}_2$, (IC2) is automatically satisfied as equality.

Proof. Use (IC3), which is binding, to rewrite (IC2) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}\leq\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}\tag{7}$$

If $\underline{w}_1<\underline{w}_2$, (7) is equivalent, given that $\bar{w}_1\geq\bar{w}_2$, to

$$\frac{e^{-a\bar{w}_2}-e^{-a\bar{w}_1}}{e^{-a\underline{w}_1}-e^{-a\underline{w}_2}}\leq\frac{1-\pi(y,\theta_1)}{\pi(y,\theta_1)}$$

But we know by (IC4) that

$$\frac{e^{-a\bar{w}_2}-e^{-a\bar{w}_1}}{e^{-a\underline{w}_1}-e^{-a\underline{w}_2}}\leq\frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

and hence, since $\pi(y,\theta_1) < \pi(y,\theta_2)$, (IC2) is slack.

If $\underline{w}_1 = \underline{w}_2$, then we know that $\bar{w}_1 = \bar{w}_2$ and (7) - hence (IC2) - is automatically satisfied. $\square$

Step 8: At an optimal solution (IC1) and (IC4) cannot be simultaneously binding if $\underline{w}_1 < \underline{w}_2$. If $\underline{w}_1 = \underline{w}_2$ they are both automatically satisfied (as equalities).

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and observe that if (IC1) and (IC4) were binding, one would have

$$\frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)} = \frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} = \frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

a contradiction. $\square$

Step 9: At an optimal solution, if $\underline{w}_1 < \underline{w}_2$ (IC4) binds.

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and (IC4) is slack and consider changing $\bar{w}_1$ and $\underline{w}_1$ by respectively $\Delta\bar{w}_1 < 0$ and $\Delta\underline{w}_1 > 0$ such that, (i) $\pi(x,\theta_1)\Delta\bar{w}_1 + (1-\pi(x,\theta_1))\Delta\underline{w}_1 < 0$ and (ii), $\pi(x,\theta_1)e^{-a(\bar{w}_1+\Delta\bar{w}_1)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1+\Delta\underline{w}_1)} = \pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1}$. Such a change exists by strict concavity of the utility function and provides higher profit to the principal.

Furthermore, this change does not affect (IC1), (IC3), and (PC) and is feasible given that (IC2), (IC4), (IC5) and (IC6) are slack. Hence, (IC4) has to be binding at an optimal solution whenever $\underline{w}_1 < \underline{w}_2$. $\square$

Steps 1-9 complete the proof of Proposition A.1. From this result it then immediately follows:

Corollary A.1: *The optimal flexible contract can be obtained as a solution to the simpler programme below:*

$$\begin{array}{ll} \max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & \quad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\ \text{s.t.} & \\ \left\{ \begin{array}{l} (IC3), (IC4), (PC) \text{ (as stated in } (P^{flex})) \text{ and} \\ (WI) \quad \bar{w}_1 \geq \bar{w}_2 \\ (WII) \quad \bar{w}_2 \geq \underline{w}_2 \end{array} \right. & \end{array} \qquad (P^{flex,R})$$

Observe the constraint $\underline{w}_2 \geq \underline{w}_1$ is implied by (WI) and $(IC4)$.

Proposition A.2: *Under Assumption 1, 2, there exists a solution to problem $(P^{flex,R})$ (and hence also to (P^{flex})).*

Proof. The two binding constraints (IC3) and (IC4) enable one to solve for $\bar{z}_1 = e^{-a\bar{w}_1}$ and $\underline{z}_1 \equiv e^{-a\underline{w}_1}$ as a function of $\bar{z}_2 \equiv e^{-a\bar{w}_2}$ and $\underline{z}_2 \equiv e^{-a\underline{w}_2}$, yielding:

$$\begin{aligned}\bar{z}_1 &= \frac{\big((1-\pi(y,\theta_2))[\pi(y,\theta_1)\bar{z}_2 + (1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c} - (1-\pi(x,\theta_1))[\pi(y,\theta_2)\bar{z}_2 + (1-\pi(y,\theta_2))\underline{z}_2]\big)}{\pi(x,\theta_1)-\pi(y,\theta_2)} \\ \underline{z}_1 &= \frac{\big(\pi(x\theta_1)[\pi(y,\theta_2)\bar{z}_2 + (1-\pi(y,\theta_2))\underline{z}_2] - \pi(y,\theta_2)[\pi(y,\theta_1)\bar{z}_2 + (1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c}\big)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\end{aligned}$$

We now want to establish that under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, it is possible to find $0 \leq \bar{z}_2 \leq \underline{z}_2$ such that:

$$\begin{aligned}\bar{z}_1 &> 0 \\ \bar{z}_1 &\leq \bar{z}_2 \\ \underline{z}_2 &\leq \underline{z}_1 \\ \bar{z}_2 &\leq \underline{z}_2\end{aligned}$$

These inequalities ensure that values of the wages satisfying $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$ can be found.
The first inequality is equivalent, under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, to

$$\frac{(1-\pi(x,\theta_1))\pi(y,\theta_2) - (1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))]} < \frac{\underline{z}_2}{\bar{z}_2} \tag{8}$$

The next two inequalities are actually equivalent (again under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$) to the same inequality:

$$\frac{\pi(x,\theta_1) - \pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))} \geq \frac{\underline{z}_2}{\bar{z}_2} \tag{9}$$

Thus, to show that we can find some values $\bar{z}_2, \underline{z}_2$ satisfying the last inequality, $\bar{z}_2 \leq \underline{z}_2$, and such that (8) and (9) hold, we need to establish that the following holds:

$$\max\left(1, \frac{(1-\pi(x,\theta_1))\pi(y,\theta_2) - (1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))]}\right) < \frac{\pi(x,\theta_1) - \pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))}$$

Straightforward computation shows that, under the assumption that $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, this is indeed the case. ■

Before solving problem $(P^{flex,R})$, observe that one can rewrite it, with the following change of variables $z = e^{-aw}$, as a problem with a (strictly) concave objective and linear constraints:

$$\max_{\bar{z}_1,\underline{z}_1,\bar{z}_2,\underline{z}_2} \quad \begin{array}{l} p[\pi(x,\theta_1)(\bar{R}+\frac{\log \bar{z}_1}{a})+(1-\pi(x,\theta_1))(\underline{R}+\frac{\log \underline{z}_1}{a})] \\ \quad +(1-p)[\pi(y,\theta_2)(\bar{R}+\frac{\log \bar{z}_2}{a})+(1-\pi(y,\theta_2))(\underline{R}+\frac{\log \underline{z}_2}{a})] \end{array}$$

$$\begin{cases} (IC3') & \pi(x,\theta_1)e^{ac_x}\bar{z}_1+(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1=\pi(y,\theta_1)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_1))e^{ac_y}\underline{z}_2 \\ (IC4') & \pi(y,\theta_2)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2=\pi(y,\theta_2)e^{ac_y}\bar{z}_1+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_1 \\ & \\ (PC') & p[\pi(x,\theta_1)e^{ac_x}\bar{z}_1+(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1]+ \\ & \qquad (1-p)[\pi(y,\theta_2)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2]\leq e^{-a\bar{u}} \\ & \\ (WI') & \bar{z}_1\leq\bar{z}_2 \\ (WII') & \bar{z}_2\leq\underline{z}_2 \end{cases} \tag{$\widetilde{P}^{flex,R}$}$$

Proposition A.3: *At a solution to the program ($\widetilde{P}^{flex,R}$), (PC') binds. Furthermore, we have that $\bar{w}_2>\underline{w}_2$.*

Proof. Consider the program ($\widetilde{P}^{flex,R}$). Let λ_3, λ_4, λ_{PC}, λ_I, and λ_{II} denote the Lagrange multipliers associated to the constraints of this problem. The first order conditions obtained by differentiating the Lagrangean with respect to $\bar{z}_1,\bar{z}_2,\underline{z}_2,\underline{z}_1$ are then:

$$\begin{cases} (i) & \frac{p\pi(x,\theta_1)}{a\bar{z}_1} = \lambda_3\pi(x,\theta_1)e^{ac_x}-\lambda_4\pi(y,\theta_2)e^{ac_y}+\lambda_{PC}p\pi(x,\theta_1)e^{ac_x}+\lambda_I \\ & \\ (ii) & \frac{p(1-\pi(x,\theta_1))}{a\underline{z}_1} = \lambda_3(1-\pi(x,\theta_1))e^{ac_x}-\lambda_4(1-\pi(y,\theta_2))e^{ac_y} \\ & \qquad +\lambda_{PC}p(1-\pi(x,\theta_1))e^{ac_x} \\ & \\ (iii) & \frac{(1-p)\pi(y,\theta_2)}{a\bar{z}_2} = -\lambda_3\pi(y,\theta_1)e^{ac_y}+\lambda_4\pi(y,\theta_2)e^{ac_y} \\ & \qquad +\lambda_{PC}(1-p)\pi(y,\theta_2)e^{ac_y}-\lambda_I+\lambda_{II} \\ & \\ (iv) & \frac{(1-p)(1-\pi(y,\theta_2))}{a\underline{z}_2} = -\lambda_3(1-\pi(y,\theta_1))e^{ac_y}+\lambda_4(1-\pi(y,\theta_2))e^{ac_y} \\ & \qquad +\lambda_{PC}(1-p)(1-\pi(y,\theta_2))e^{ac_y}-\lambda_{II} \end{cases}$$

Multiplying each equation by the appropriate z variable, adding the four equations of the above system and using the fact that ($IC3'$) and ($IC4'$), in the above specification of the optimization problem, are written as equalities, yields the following:

$$\begin{aligned} \frac{1}{a} = {} & \lambda_{PC}[p\pi(x,\theta_1)e^{ac_x}\bar{z}_1+p(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1 \\ & +(1-p)\pi(y,\theta_2)e^{ac_y}\bar{z}_2+(1-p)(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2]+\lambda_I[\bar{z}_1-\bar{z}_2]+\lambda_{II}[\bar{z}_2-\underline{z}_2] \end{aligned}$$

Using the complementarity slackness condition, we get that $\lambda_I[\bar{z}_1-\bar{z}_2]=\lambda_{II}[\bar{z}_2-\underline{z}_2]=0$. Hence $\lambda_{PC}>0$, which establishes that (PC') binds. Hence, we can conclude from the expression above that $\lambda_{PC}=\frac{e^{a\bar{u}}}{a}$.

Next we want to show that $\bar{w}_2 > \underline{w}_2$ or equivalently $\underline{z}_2 > \bar{z}_2$. Assume to the contrary that $\bar{z}_2 = \underline{z}_2 \equiv z_2$. We know in that case that (WI') is slack (otherwise by $(IC4')$ all wages would have to be equal, but this would contradict the fact that $(IC3')$ binds) and hence $\lambda_I = 0$. Rewrite now FOC's (iii) and (iv) as:

$$\begin{cases} (iii) & \frac{(1-p)}{az_2} = -\lambda_3 \frac{\pi(y,\theta_1)}{\pi(y,\theta_2)} e^{ac_y} + \lambda_4 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} + \frac{\lambda_{II}}{\pi(y,\theta_2)} \\ (iv) & \frac{(1-p)}{az_2} = -\lambda_3 \frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)} e^{ac_y} + \lambda_4 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} - \frac{\lambda_{II}}{1-\pi(y,\theta_2)} \end{cases}$$

This implies that

$$-\lambda_3 \frac{\pi(y,\theta_1)}{\pi(y,\theta_2)} e^{ac_y} + \frac{\lambda_{II}}{\pi(y,\theta_2)} = -\lambda_3 \frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)} e^{ac_y} - \frac{\lambda_{II}}{1-\pi(y,\theta_2)}$$

or, after some simplification,

$$\lambda_{II} = (\pi(y,\theta_1) - \pi(y,\theta_2))\lambda_3 e^{ac_y}$$

Note that $(\pi(y,\theta_1) - \pi(y,\theta_2)) < 0$ and hence $\lambda_{II} \geq 0$ iff $\lambda_3 \leq 0$. Next observe that (PC') as an equality together with $(IC3')$ imply, if $\bar{z}_2 = \underline{z}_2 \equiv z_2$, that $z_2 = e^{-a(c_y+\bar{u})}$. Plug now the values of λ_{PC} and z_2 into equations (iii) and (iv) and use the expression for λ_{II} obtained above. The two equations are identical and yield $\lambda_4 = \lambda_3 \equiv \lambda$.

We have so a system of four equations – FOC's (i) and (ii), $(IC3')$ and $(IC4')$ – to determine three variables: λ, $\bar{z}_1$ and $\underline{z}_1$. $(IC3')$ and $(IC4')$ can be used to solve directly for $\bar{z}_1$ and $\underline{z}_1$. Now, the two FOC's can be rewritten:

$$\begin{aligned} p\pi(x,\theta_1) &= a\lambda\bar{z}_1(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y}) + e^{a\bar{u}}\bar{z}_1 p\pi(x,\theta_1)e^{ac_x} \\ p(1-\pi(x,\theta_1)) &= a\lambda\underline{z}_1((1-\pi(x,\theta_1))e^{ac_x} - (1-\pi(y,\theta_2))e^{ac_y}) + e^{a\bar{u}}\underline{z}_1 p(1-\pi(x,\theta_1))e^{ac_x} \end{aligned}$$

Adding these two equations yields an equation

$$\begin{aligned} p &= a\lambda\left[\bar{z}_1(\pi(x,\theta_1)e^{ac_x} + \underline{z}_1(1-\pi(x,\theta_1))e^{ac_x} - \pi(y,\theta_2)\bar{z}_1 e^{ac_y} - \underline{z}_1(1-\pi(y,\theta_2))e^{ac_y}\right] + \\ &\quad + pe^{a\bar{u}}\left[\bar{z}_1\pi(x,\theta_1)e^{ac_x} + \underline{z}_1(1-\pi(x,\theta_1))e^{ac_x}\right] \end{aligned}$$

which, using $(IC3')$ and $(IC4')$ can be rewritten as:

$$p = a\lambda\left[e^{ac_y}e^{-a(c_y+\bar{u})} - e^{ac_y}e^{-a(c_y+\bar{u})}\right] + pe^{a\bar{u}}\left[e^{ac_y}e^{-a(c_y+\bar{u})}\right] = p\left[e^{ac_y}e^{-ac_y}\right] = p$$

always satisfied, so that one of the two above equations can be dropped. The remaining one can be used to solve for λ. Recall that $\lambda \leq 0$ is needed to ensure that $\lambda_{II} \geq 0$.

Solving then $(IC3')$ and $(IC4')$ with respect to $\bar{z}_1$ and $\underline{z}_1$ we get:

$$\underline{z}_1 = \frac{\pi(x,\theta_1)e^{-a(c_y+\bar{u})} - \pi(y,\theta_2)e^{-a(c_x+\bar{u})}}{\pi(x,\theta_1) - \pi(y,\theta_2)}$$
$$\bar{z}_1 = \frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1) - \pi(y,\theta_2)}.$$

Substituting into the first of the two FOC's above yields:

$$p\pi(x,\theta_1) = [a\lambda(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y}) + e^{a\bar{u}}p\pi(x,\theta_1)e^{ac_x}] \cdot \cdot \frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1) - \pi(y,\theta_2)}$$

and hence

$$\begin{aligned} & a\lambda(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y})\frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1) - \pi(y,\theta_2)} \quad (10) \\ = \ & p\pi(x,\theta_1) - p\pi(x,\theta_1)e^{ac_x}\frac{(1-\pi(y,\theta_2))e^{-ac_x} - (1-\pi(x,\theta_1))e^{-ac_y}}{\pi(x,\theta_1) - \pi(y,\theta_2)} = \\ = \ & p\pi(x,\theta_1)\left[\frac{(1-\pi(x,\theta_1))e^{a\Delta c} - (1-\pi(y,\theta_2)) + \pi(x,\theta_1) - \pi(y,\theta_2)}{\pi(x,\theta_1) - \pi(y,\theta_2)}\right] = \\ = \ & p\pi(x,\theta_1)\left[\frac{(1-\pi(x,\theta_1))(e^{a\Delta c} - 1)}{\pi(x,\theta_1) - \pi(y,\theta_2)}\right] > 0 \end{aligned}$$

Since the coefficient of λ in the first term is positive, it follows that the solution for λ of such equation is > 0, a contradiction. Hence, it cannot be that $\bar{z}_2 = \underline{z}_2$. ■

This completes the proof of Proposition 1.■

Proof of Proposition 2.

The first best optimal contract is obtained as solution of the problem of maximizing the principal's expected profits subject to the agent's participation constraint, which under risk neutrality takes the following form:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R} - \bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R} - \underline{w}_1)] + (1-p)[\pi(y,\theta_2)(\bar{R} - \bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R} - \underline{w}_2)]$$

$$\text{s.t.} \quad p[\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x] + (1-p)[\pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y] \geq \bar{u}$$

The maximal level of the principal's expected profits that can be attained at a solution of this problem is then clearly the one stated in the proposition and it is immediate to verify that the compensation profile given in (1) yields such level of expected profits and is then a first best optimum. It remains thus to verify the values in (1) satisfy all the incentive compatibility constraints, which under risk neutrality take the following form:

$$\left\{\begin{array}{l} \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 \geq \pi(x,\theta_1)\bar{w}_2 + (1-\pi(x,\theta_1))\underline{w}_2 \\ \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_1 + (1-\pi(y,\theta_1))\underline{w}_1 - c_y \\ \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_2 + (1-\pi(y,\theta_1))\underline{w}_2 - c_y \\ \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 \geq \pi(y,\theta_2)\bar{w}_1 + (1-\pi(y,\theta_2))\underline{w}_1 \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_2 + (1-\pi(x,\theta_2))\underline{w}_2 - c_x \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_1 + (1-\pi(x,\theta_2))\underline{w}_1 - c_x \end{array}\right. \tag{11}$$

This is immediate by direct substitution. $\square$

Proof of Proposition 3.

We first consider a local deviation from the contract specified in (4) such that $d\underline{w}_1 > 0$ and such that $(IC3^*)$, $(IC4^*)$ and (PC^*) continue to hold as equalities. We conjecture, and verify below, that the sign of the changes in the other wage variables is as follows, $d\bar{w}_1 < 0$, $d\bar{w}_2 > 0$, and $d\underline{w}_2 < 0$, and in the Agent's expected utility in the two θ states is $du(\theta_1) < 0$, $du(\theta_2) > 0$. That is, the Agent is no longer fully insured in state θ_2 nor across states θ_1 and θ_2, which fixes his "beliefs" in the incentive and participation constraints.

Differentiating $(IC3^*)$, $(IC4^*)$ and (PC^*), written as equalities, with respect to $\bar{w}_1, \underline{w}_1, \bar{w}_2, \underline{w}_2$, and solving these equations for $d\bar{w}_1$, $d\bar{w}_2$, $d\underline{w}_2$, as a function of $d\underline{w}_1 > 0$ yields:

$$\begin{aligned} d\bar{w}_1 &= \left[\frac{-1}{(\hat{p}+\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)) + (1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))} + 1\right] d\underline{w}_1 \\ d\bar{w}_2 &= \left[1 - \frac{\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)+[\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)-\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1)][\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)]}{[(\hat{p}+\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)) + (1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))][\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)]}\right] d\underline{w}_1 \\ d\underline{w}_2 &= \left[1 - \frac{[\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)-\hat{\pi}(x,\theta_1)+\alpha(x,\theta_1)][\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)]}{[(\hat{p}+\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)) + (1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))][\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)]}\right] d\underline{w}_1 \end{aligned} \tag{12}$$

It is immediate to verify from the above expressions that the sign of the changes is the one conjectured.

The change in the Principal's profit is given by

$$-\{\hat{p}\,[\hat{\pi}(x,\theta_1)d\bar{w}_1 + (1-\hat{\pi}(x,\theta_1))d\underline{w}_1] + (1-\hat{p})\,[\hat{\pi}(y,\theta_2)d\bar{w}_2 + (1-\hat{\pi}(y,\theta_2))d\underline{w}_2]\}$$

Substituting for $d\bar{w}_1$, $d\bar{w}_2$, $d\underline{w}_2$ the expressions found in (12) yields:

$$\begin{aligned} \{ -1 &+ \frac{\hat{p}\hat{\pi}(x,\theta_1)+(1-\hat{p})\hat{\pi}(y,\theta_2)}{[(\hat{p}+\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)) + (1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))]} \\ &+(1-\hat{p})\frac{[\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_1)+\alpha(y,\theta_1)]\alpha(y,\theta_2)}{[(\hat{p}+\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)) + (1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))][\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)-\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)]} \} d\underline{w}_1 \end{aligned}$$

The sign of this expression can be positive or negative, depending on the parameters of the model, as claimed in the text: it is positive when $\alpha(p) = \alpha(y, \theta_2) = 0$, $\alpha(x, \theta_1) > 0$ and it is negative when $\alpha(x, \theta_1) = 0$.

The other possible deviation, with $d\underline{w}_1 < 0$, can be treated in a similar fashion. The wage changes have here the opposite sign as above, hence the induced beliefs need to be modified accordingly. The expression for the change in expected profits in that case is then:

$$\begin{aligned}
&\{-1-\frac{\hat{p}\hat{\pi}(x,\theta_1)+(1-\hat{p})\hat{\pi}(y,\theta_2)}{[(\hat{p}-\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))+(1-\hat{p}+\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))]}\\
&-(1-\hat{p})\frac{\hat{\pi}(y,\theta_2)[\hat{\pi}(y,\theta_1)+\alpha(y,\theta_1)-2\alpha(y,\theta_2)-\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)]}{[(\hat{p}-\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))+(1-\hat{p}+\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))][\hat{\pi}(y,\theta_1)+\alpha(y,\theta_1)-\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)]}\\
&-(1-\hat{p})\frac{-(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))(\hat{\pi}(y,\theta_1)+\alpha(y,\theta_1))+(\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))}{[(\hat{p}-\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))+(1-\hat{p}+\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))][\hat{\pi}(y,\theta_1)+\alpha(y,\theta_1)-\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)]}\}d\underline{w}_1
\end{aligned}$$

The Principal would benefit from this deviation only if the term appearing in curly brackets is negative (as in this case $d\underline{w}_1 < 0$). This term is negative if and only if

$$\begin{aligned}
&(\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)-\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1))\times\\
&[(\hat{p}-\alpha(p))(\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1))+(1-\hat{p}-\alpha(p))(\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2))]-\\
&(1-\hat{p})\alpha(y,\theta_2)[\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1)]>0
\end{aligned}$$

It can be shown that the expression on the left hand side of the above inequality is bounded above by

$$-\alpha(x,\theta_1)\hat{p}(\hat{\pi}(y,\theta_2)+\alpha(y,\theta_2)-\hat{\pi}(y,\theta_1)-\alpha(y,\theta_1))-(1-\hat{p})\alpha(y,\theta_2)[\hat{\pi}(x,\theta_1)-\alpha(x,\theta_1)-\hat{\pi}(y,\theta_2)-\alpha(y,\theta_2)]$$

which is always negative. Hence, the considered deviation is never optimal.■